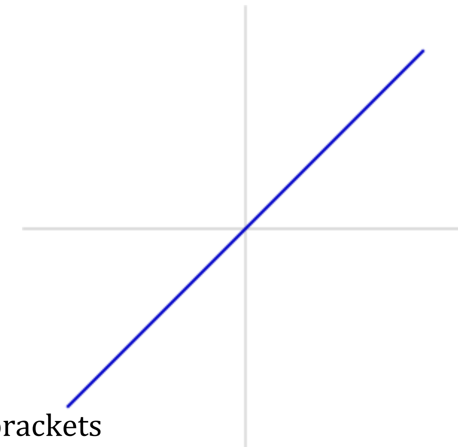
Introduction to Functions:



Function Notation:

This uses '$f(x)$' instead of 'y'

Know how to solve an equation

Example in a Linear Function:
$f(x) = ax + b$
$f(-2) = a(-2) + b$

Essentially replace 'x' with the number within the brackets

Is it a Function?

Use the vertical line test:
If the line passes through the line once or more, it is not a function
If it passes through once, it is a function

Domain and Range:

Domain:

All possible x values you can plug into the equation.
How far left and right does it go?

Notation used:

$x \neq a, x \geq b, x \leq c, x \in \mathbb{R}$

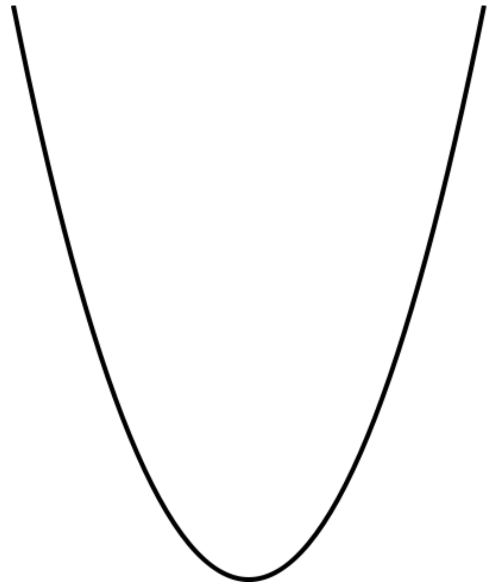
This means that:
x cannot equal to 'a'
x can be equal or greater than 'b'
x can be equal or smaller than 'c'
x is an **element** of all **real numbers** or '$\mathbb{R}$'

Range:

All possible y values you can plug into the equation.
How far up and down does it go?

Same notations apply for y as for x.

Parabolas:



Different Ways to Express a Parabola:

$y = ax^2 + bx + c$

$y = a(x - r)(x - s)$

$y = a(x - h)^2 + c$

How to Find the Vertex:

$x = \frac{-b}{2a}$
(This takes its values from the 1st equation)

Example:

$f(x) = \frac{1}{2}x^2 - 6x + 13$
$x = \frac{-(-6)}{2(\frac{1}{2})}$
$x = 6$
Plug in x into the equation and solve for y:
$y = -5$
$\therefore\ vertex\ is\ (6, -5)$

How to Solve a Quadratic Function: (or finding the x-intercept)

- ***Using Factoring:***

Take this form:

$y = ax^2 + bx + c$

Then move everything to one side so that:

$0 = ax^2 + bx + c$

Factor it:

$0 = a(x + r)(x + s)$

This means that:

$x + r = 0\ or\ x + s = 0$

$x = -3\ or\ x = -3$

The x-intercept is -3

How to factor:

1) Pull out any number/letter that appears in **all** terms.
2) Does it start with x^2 or with sx^2, where 's' is a random number?

    a. If you get x^2

        i. Find 2 numbers that add to 'b' and multiply to make 'c'

    b. If you get sx^2

        i. Find 2 numbers that add to 'b' and multiply to make 'a'

        ii. "Break up" the middle term using those 2 numbers (let's call them 'q' and 'w') so it looks like:

        $qx^2 + wx + qx + w$

        iii. Factor the first 2 and the second 2:

        $x(qx + w) + (qx + w)$

        iv. Pull out brackets:

        $(qx + w)(x + 1)$

- ***Quadratic Formula***

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Fill the values in and solve for x.
Note that there is a $\pm$ in the equation. Therefore there is a possibly for two values to come out of this.

Find the Intersection(s) of a Line and a Parabola:

1. Set each equal to each other
2. Move it all to one side
3. Foil/simplify/etc.
4. Solve with factoring or quadratic function
5. Plug each x value into either original equation
6. State answer in co-ordinate form

Find the Equation of a Parabola Using 3 Points:

1. Using the x-intercepts, make a factorized equation
2. To find 'a' in the aforementioned equation, use the 3rd part and solve for 'a'

Find the Inverse of an Equation:

1. Switch the x and y in your function
2. Isolate the y value (e.g. y = …)

Note: Don't forget the ± when square rooting

Inverse of Parabolas:

1. Switch x with y
2. Solve for y so that y=[…]

Linear – Quadratic Systems: (Intersections of Line and Parabola)

Find where w(x), a line, and g(x), a parabola, intersect.

$w(x) = ax + b$

$g(x) = c(x - d)(x + e)$

1) Set the two equations equal to each other

   $ax + b = c(x + d)(x + e)$

2) Move everything to one side

   $0 = c(x + d)(x + e) + ax + b$

   $0 = c(x^2 + (d + e)x + de) + ax + b$

   $0 = cx^2 + cx(d + e) + ax + dec + b$

   (Solve for y)

3) Factor or use the Quadratic Formula (both covered above) to solve for x (you may find 0, 1 or 2 values).

   $y = c(x + d)(x + e)$

   $y = cx^2 + c(d + e)x + dec$

4) Plug each x value into either of the original equations

5) State answer (in coordinate form): (x,y)

Families of Quadratic Parabolas:

Find the equation of a parabola with x-intercept using 3 points. 2 points are on the x-axis (x-intercepts) and one other point on the parabola.

Example: x-intercepts of a and b with the point (c,d)

1) Make the equation:

$y = a(x - a)(x - b)$

2) Plug in the 3rd point:

$d = a(c - a)(c - b)$

3) Solve for 'a'

Note that if there is only <u>one</u> x-intercept, both parentheses are the same.

Example: If the x-intercept is 7, y=(x-7)(x-7)

Sketching Parabolas:

1) Find the Vertex
2) Then use the step pattern:

What is the step pattern?

1,3,5,7 multiplied by 'a'

Example:

$f(x) = -2(x + 3)^2 + 6$

1) Vertex: (-3, 6)

-3 from the horizontal shift, or 'h'
6 from the vertical shift, or 'c'

2) Step pattern: -2, -6, -10, -14

'a' multiplied by each number of the step pattern

Reciprocal Functions:

General Equation:

$$y = \frac{a}{x - b} + c$$

a – the stretch
b – the vertical asymptote
c – the horizontal asymptote

How to graph a function:

1. Take $x - b = 0$, solve for x.
2. That value of x is the vertical asymptote
3. When x approaches ∞ (or becomes a really

big number), what does y become?

4. That value of y is the vertical asymptote
5. Use random values on both sides of the

vertical asymptote.

How to find the equation of a reciprocal function:

1. Find the asymptotes of the function
2. Put them into the general equation (at the begging of the section)
3. Find a point on the function and solve for a

Trigonometry

Pythagorean Theorem:

$$a^2 + b^2 = c^2$$

Note that this theorem only applies in right-angle triangles.

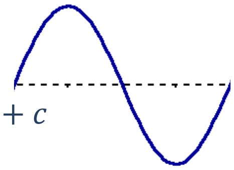
Sine Law:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine Law:

$$c^2 = a^2 + b^2 + 2ab\cos C$$

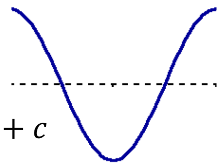
Sine Wave:



$y = a\,sin[k(x-h)] + c$

a – Amplitude
k – Alters the period
h – Horizontal shift
c – Vertical Shift

Cosine Wave:



$y = a\cos[k(x-h)] + c$

a – Amplitude
k – Alters the period
h – Horizontal shift
c – Vertical Shift

Note that the period (or p) is $p = \frac{360}{k}$

Arithmetic Sequences:

Determine a Term in the Sequence:

$t_n = a + (n-1)d$

t_n – the n^{th} term
a – the initial value
n – the term number
d – the common difference

The Sum: (to the n^{th} term)

$S_n = \frac{n}{2}(a + t_n)$

S_n – the sum of the terms
n – number of terms you are finding the sum of
a – the first term of the sum
t_n – the final term of the sum

Geometric Sequences:

Determine a Term in a Sequence:

$t_n = a(r)^{(n-1)}$

t_n – the n^{th} term
a – the first term of the sequence
r – the common ratio
n – the term number

The Sum: (to the n^{th} term)

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_n = \frac{ar^n - a}{r - 1}$$

$$S_n = \frac{t_{n+1} - a}{r - 1}$$

S_n – the sum of the terms
a – the first term of the sequence
r^n – common ratio to the power of the final term number
r – common ratio

Simple vs. Compound Interest

Simple Interest:

Interest that collects only on the initial investment.

100 dollars at 5% per annum:
5 \$ the first year
5 \$ the second year

How to find out how much interest is gained:

$I = Prt$

I – Interest Earned
P – Principle (starting investment)
r – Interest Rate
t – Time elapsed

Compound Interest:

Interest the collects on the initial investment <u>and</u> the interest you've already earned.

100 dollars at 5% per annum:
5 $ the first year
5.03 $ the second year

How to Calculate Amount of Money in Account

$A = P(1 + i)^n$

A – Amount in account
P – Principle (amount of money at the start of the investment)
i – Interest rate per compounding period
n – Number of compounding periods elapsed

$$i = \frac{interest\ rate}{compound\ period}$$

Compound periods:

Annually – Once per year
Semiannually – Twice per year
Quarterly – Four times per year
Bimonthly – Every 2 months
Monthly – Every month
Weekly – 52x per year
Daily – 365x per year

Annuities:

Note that for this subsection:

r = the yearly nominal interest rate.
t = the number of years.
m = the number of periods per year.
i = the interest rate per period.
n = the number of periods.

$$\mathrm{i} = \frac{\mathrm{r}}{\mathrm{m}}$$

$$\mathrm{n} = \mathrm{t} \cdot \mathrm{m}$$

If you are looking for (or know) the future value (written as 'S') of a annuity:

$$S = \frac{R[(1+i)^n - 1]}{i}$$

Misc.:

Change of Base:

$$\log_n m = \frac{\log_x m}{\log_x n}$$