

MCR3U – Test 3 (Exponential Functions)

K / 17.5
T / 8.5
C / 8
A / 13
Total / 46

Name: Answers

1. Simplify each as much as possible, and give your answer with only positive exponents.

a) $(r^4 r^{-2})^{-\frac{1}{2}} = (r^2)^{-\frac{1}{2}} = r^{-1} = \frac{1}{r}$

b) $\left(\frac{2x}{3}\right)^{-2} = \left(\frac{3}{2x}\right)^2 = \frac{9}{4x^2}$

6 K

c) $\frac{s^{\frac{5}{6}} s^{\frac{-1}{6}}}{s^{\frac{1}{3}}} = \frac{s^{\frac{4}{6}}}{s^{\frac{1}{3}}} = s^{\frac{1}{3}}$

d) $(2x^{\frac{1}{5}})^7 (x^{\frac{1}{5}})^9 = 2^7 x^{\frac{7}{5}} x^{\frac{9}{5}} = 128 x^{\frac{16}{5}}$

2. Evaluate the following, and give your answer as an **integer or fraction** (do not have exponents in your answer!)

a) $\left(\frac{9}{7}\right)^{-2} = \frac{49}{81}$

b) $\left(\frac{27}{-64}\right)^{-\frac{4}{3}} = \left(\sqrt[3]{\frac{-64}{27}}\right)^4 = \left(\frac{-4}{3}\right)^4 = \frac{256}{81}$

4 K

3. Give the co-ordinates of each of the following for $y = -\left(\frac{1}{3}\right)^{\frac{1}{4}(x-2)} + 7$

New origin: $(2, 7)$

1st point $(2, 6)$

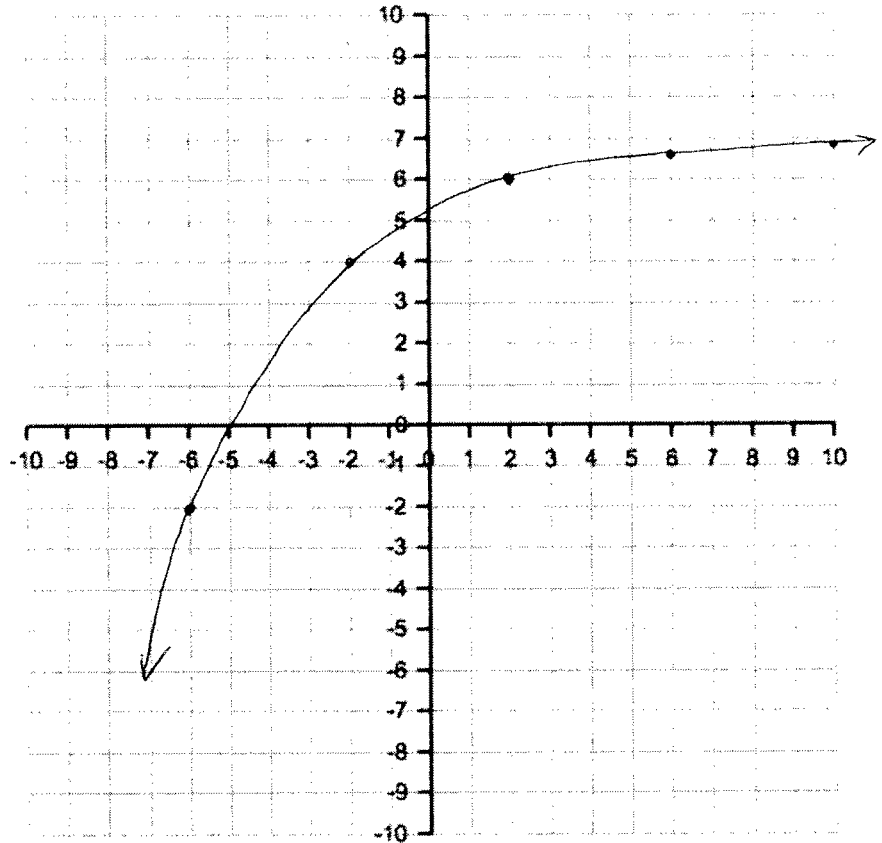
a 2nd point: $(6, 6^{\frac{2}{3}})$

a 3rd point: $(10, 6^{\frac{3}{2}})$

also $(-2, 4)$
 $(-6, -2)$

4 A

4. Now, graph the function.



5. Using exponent rules, prove that the graph of $y = 2^{x-3} + 5$ is the same as the graph of $y = \left(\frac{1}{4}\right)^{6-2x} + 5$.

6. Using exponent rules, prove that the graph of $y = 2(16)^{x+2} + 8$ is the same as the graph of $y = (8)2^{4x} + 5$.

7. The half-life of a particular radioactive isotope is 54 hours.

$$A = A_0 \left(\frac{1}{2} \right)^{t/h}$$

- a) Write an equation to relate the mass of radioactive material remaining to time (in hours).

$$A = A_0 \left(\frac{1}{2} \right)^{t/54}$$

1 K

- b) Starting with 170 g, what mass will remain in 1 week?

$$A = (170) \left(\frac{1}{2} \right)^{(24 \times 7)/54}$$

$$= 19.67 \text{ g}$$

2 K

- c) If, after one day (24 hours), a different sample of the material was found to have a mass of 14 mg, what was the initial mass of the sample?

$$14 = A_0 \left(\frac{1}{2} \right)^{24/54}$$

$$A_0 = \frac{14}{0.73487}$$

$$= 19.05 \text{ mg}$$

2 K

8. Yesterday, Mr. Oldridge's girlfriend bought a condo (true story!) for \$310,000 (number changed for privacy reasons).

- a) Assuming that the condo increases in value by 8% each year, how much will it be worth in four years?

$$V = 310000(1.08)^4$$
$$= \$421,751.58$$

2 A

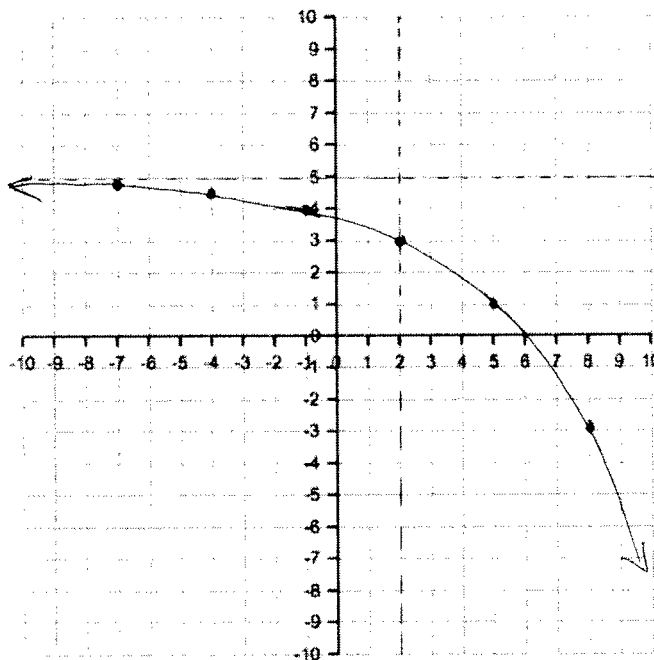
- b) Assuming that the condo **decreases** in value by 4% each year, how much will it be worth in four years?

$$V = 310000(0.96)^4$$
$$= \$263,297.43$$

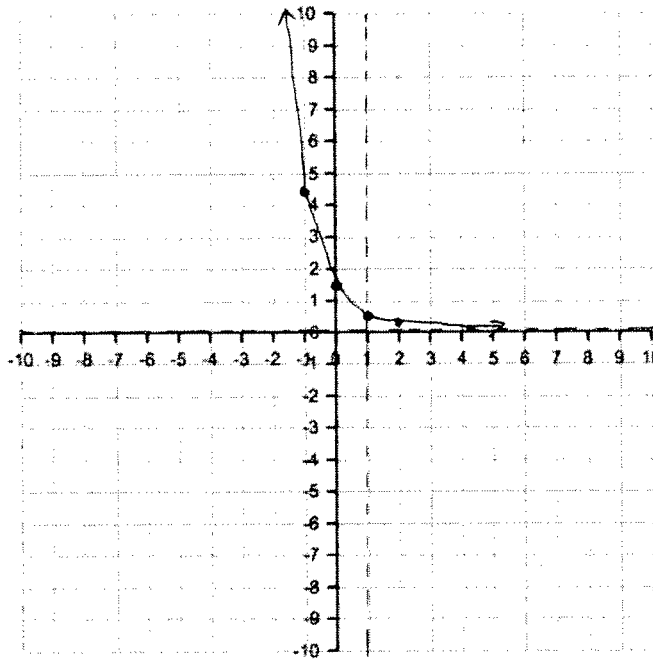
2 A

9. Graph each of the following functions.

$$f(x) = -2(2)^{\frac{1}{3}(x-2)} + 5$$



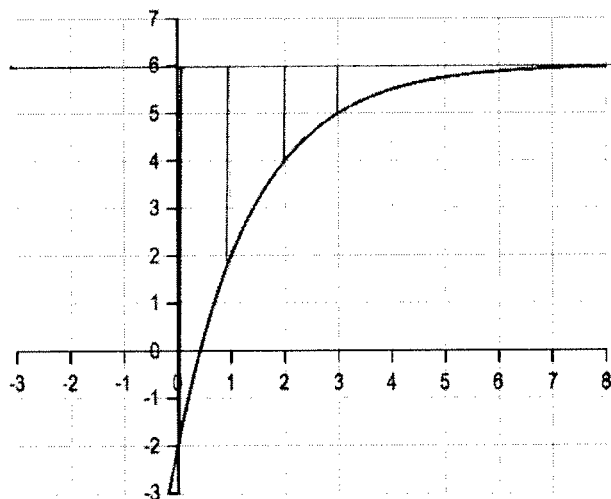
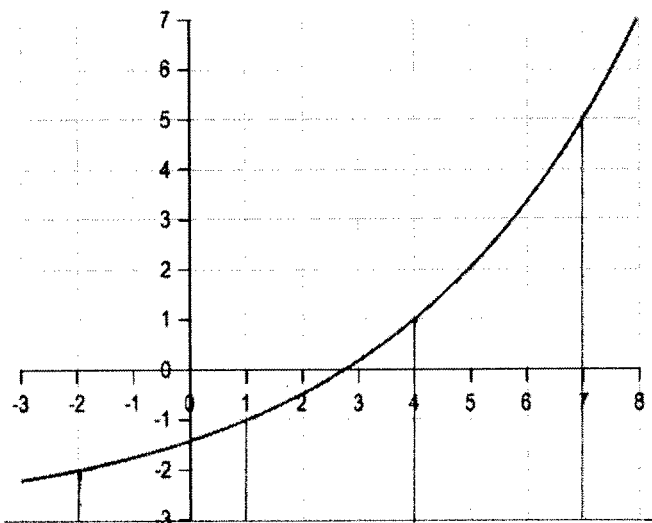
$$g(x) = \frac{1}{2}(3)^{(1-x)} = \frac{1}{2}(3)^{-(x-1)}$$



10. Give a valid equation for each of the following functions:

a) $y = 2^{\frac{x+2}{3}} - 3 = 4(2)^{\frac{x-4}{3}} - 3$
 $= 2(2)^{\frac{x-1}{3}} - 3 = 8(2)^{\frac{x-7}{3}} - 3$

b) $y = -(\frac{1}{2})^{x-3} + 6$
 $= -2(\frac{1}{2})^{x-2} + 6$
 $= -4(\frac{1}{2})^{x-1} + 6 = -8(\frac{1}{2})^x + 6$



11. Write **two** equivalent exponential expressions for each of the following.

a) $h(x) = 5\left(\frac{1}{5}\right)^{\frac{x-5}{7}} - 7$

$$= 5(5)^{-\frac{1}{7}(x-5)} - 7$$

$$= 5^{-\frac{1}{7}(x-12)} - 7$$

b) $j(x) = -(16)^{2-x} + 4$

$$= -\left(\frac{1}{4}\right)^{x-2} + 4$$

$$= -\left(\frac{1}{4}\right)^{x-2} + 4$$

$$= -\left(\frac{1}{2}\right)^{4(x-2)} + 4$$

$$= -\left(\frac{1}{2}\right)^{4(x-2)} + 4$$

$$= -16\left(\frac{1}{4}\right)^x$$

