

/17K	/8T	/5C	/10A	Total	/40
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Name: Answers Date: _____

MCR3U

Chapter 6 and 7 Test: Sequences, Series and Financial Math

/3
K

1. Using an appropriate formula for t_n , calculate the 12th term of the sequence $\{ 5, 10, 20, 40, \dots \}$.

$$a = 5$$

$$r = 2$$

$$t_{12} = (5)(2)^{12-1}$$

$$= 10240$$

/3
K

2. Using an appropriate formula for t_n , calculate the 45th term of the sequence $\{ -90, -83, -76, -69, \dots \}$

$$a = -90$$

$$d = 7$$

$$t_{45} = -90 + (45-1)(7)$$

$$= 218$$

1/3
A

3. The number 546 occurs as a term in the sequence $\{-65, -52, -39, -26, \dots\}$. Which term is it? (Put another way, for what n does $t_n = 546$?) Use an appropriate t_n formula.

$$a = -65$$

$$d = 13$$

$$546 = -65 + (n-1)(13)$$

$$\frac{546 + 65}{13} = \frac{(n-1)(13)}{13}$$

$$47 = n-1$$

$$n = 48$$

1/3
A

4. The number 6561 occurs as a term in the sequence $\{\frac{1}{9}, \frac{1}{3}, 1, 3, \dots\}$. Which term is it? (Put another way, for what n does $t_n = 6561$?) Use an proper t_n formula.

$$a = \frac{1}{9}$$

$$r = 3$$

$$\frac{6561}{(\frac{1}{9})} = \frac{(\frac{1}{9})(3)^{n-1}}{(\frac{1}{9})}$$

$$59049 = 3^{n-1}$$

$$n-1 = \frac{\log 59049}{\log 3}$$

$$n-1 = 10$$

$$n = 11$$

1/4
T

5. Find the general term ($t_n = \dots$) for an arithmetic sequence where $t_9 = 30$ and $t_{22} = 108$.

$$30 = a + (9-1)d$$

$$30 = a + 8d$$

$$108 = a + (22-1)d$$

$$108 = a + 21d$$

$$-78 = -13d$$

$$d = 6$$

$$\text{so } 30 = a + 8(6)$$

$$a = -18$$

$$\therefore t_n = -18 + (n-1)(6)$$

1/4
T

6. Find the general term ($t_n = \dots$) for an geometric sequence where $t_7 = 5.76$ and $t_{14} = 737.28$.

$$5.76 = ar^{7-1}$$

$$737.28 = ar^{14-1}$$

$$0.0078125 = r^{6-13} = r^{-7}$$

$$r = \sqrt[7]{0.0078125}$$

$$r = 2$$

$$5.76 = a(2)^6$$

$$\therefore t_n = 0.09(2)^{n-1}$$

$$a = \frac{5.76}{2^6} = 0.09$$

/3
K7. Using an appropriate formula for S_n , calculate the sum of:

$$-64 - 57 - 50 - 43 - \dots + 90 + 97$$

$$a = -64$$

$$d = 7$$

$$S_{24} = \frac{24}{2}(-64 + 97)$$

$$= 12(33)$$

$$= 396$$

$$97 = -64 + (n-1)7$$

$$\frac{97+64}{7} = n-1$$

$$n-1 = 23$$

$$n = 24$$

/3
K8. Using an appropriate formula for S_n , calculate the sum of:

$$1024 + 512 + 256 + \dots + \frac{1}{8} + \frac{1}{16}$$

$$a = 1024$$

$$r = \frac{1}{2}$$

$$S_n = \frac{1024(0.5^{15} - 1)}{0.5 - 1}$$

$$= 2047.9375$$

$$= 2047 \frac{15}{16}$$

$$\frac{1}{16} = (1024 \times \frac{1}{2})^n$$

$$n-1 = \frac{\log\left(\frac{1}{16(1024)}\right)}{\log \frac{1}{2}}$$

$$n-1 = 14$$

$$n = 15$$

/2
A

9. Your stock portfolio has had a great year and you decide to sell them all. You get \$4,000 and invest it in a savings account that pays 4%/year, simple interest. How much money will be in the account after 10 years?

$$~~A = 4000(1 + 0.04)^{10}~~$$

$$I = 4000(0.04)(10) \\ = 1600 \quad \checkmark$$

$$A = 5600 \quad \checkmark$$

/2
A

10. How much money would be in an account 10 years later, if you invest \$4,000 in an account collects 4%/year, compounded monthly?

$$A = 4000 \left(1 + \frac{0.04}{12}\right)^{10 \times 12} \\ = 5963.33$$

/2
C

11. All else being equal, which pays more ... simple interest or compound interest? What is the difference between how compound and simple interest are calculated, that leads to this difference?

Compound

This way, you collect interest on the interest you already collected

12. You (16 years old) decide to start a retirement savings plan. This will involve ^{weekly} ~~monthly~~ deposits of \$200 into an account earning 5.2% interest compounded weekly.

- $\frac{1}{3}$ K a) Determine the value of the investment when you retire at 60 years old.

$$A = \frac{200 \left[\left(1 + \frac{0.052}{52} \right)^{44 \times 52} - 1 \right]}{\left(\frac{0.052}{52} \right)}$$

$$= \$1,768,789.43$$

- $\frac{1}{2}$ K b) How much interest will be earned in total?

$$\text{Total invested: } (44 \times 52) \times 200 = 457600$$

$$I = 1768789 - 457600 = \$1,311,189$$

- $\frac{1}{3}$ A 13. What regular monthly payment is needed to have \$20,000 in 10 years, if you're earning 8%/year, compounded monthly?

$$20000 = \frac{R \left[\left(1 + \frac{0.08}{12} \right)^{120} - 1 \right]}{\left(\frac{0.08}{12} \right)}$$

$$133.33 = R (1.2196)$$

$$R = \$109.32$$

Arithmetic Sequence: $t_n = a + (n-1)d$

Geometric Sequence: $t_n = ar^{n-1}$

Arithmetic Series: $S_n = \frac{n}{2}(a + t_n)$

Geometric Series: $S_n = \frac{a(r^n - 1)}{r - 1}$

Simple Interest: $I = Prt$

Compound Interest: $A = P(1+i)^n$

Annuities: $A = \frac{R[(1+i)^n - 1]}{i}$