

# MCR3U – Unit 4 (Trigonometry) Test

Show all related work.

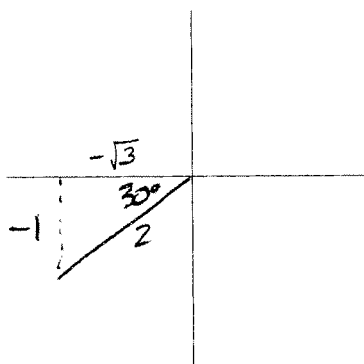
Include units if applicable.

Name: Answer

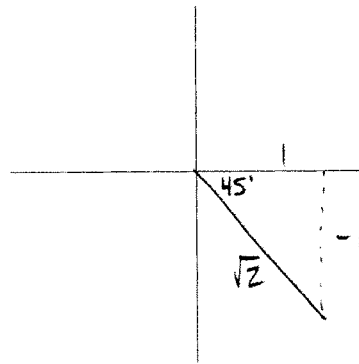
16 K 11 T 5 C 14 A 46 Total

1. Please give the **exact** trigonometric ratios that correspond to the following angles. Include a sketch of the angle!

a)  $\tan 210^\circ = \boxed{\frac{-1}{-\sqrt{3}}}$

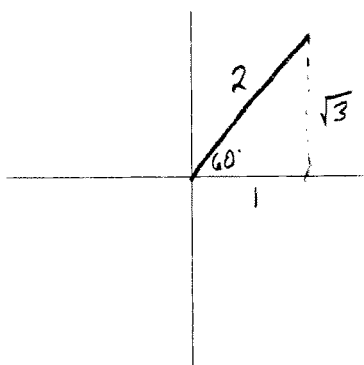


b)  $\cos 315^\circ = \boxed{\frac{1}{\sqrt{2}}}$

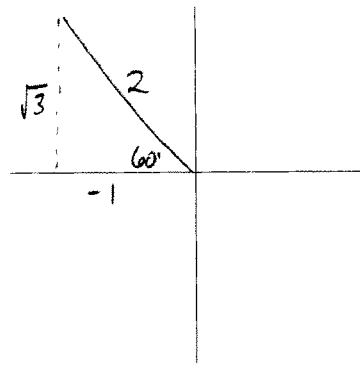


4 K

c)  $\sin 60^\circ = \boxed{\frac{\sqrt{3}}{2}}$

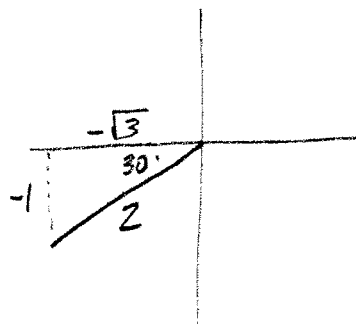
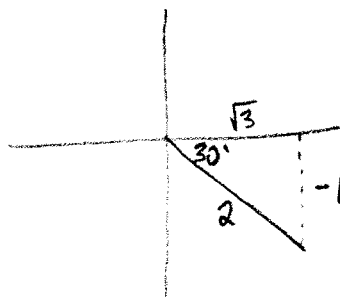


d)  $\tan 120^\circ = \boxed{\frac{\sqrt{3}}{-1}}$



1 C

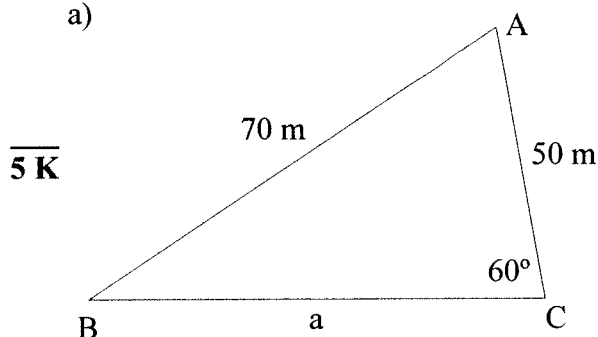
2. Explain how it's possible that  $\sin 330^\circ$  and  $\sin 210^\circ$  give identical exact trigonometric ratios.



Same opposite  
Same hypotenuse

2. Please solve the following triangles FULLY. That is, give values for each of the following angles and sides. Please show ALL of your work, and **don't forget UNITS**:

a)



Angle A: 81.79°

Angle B: 38.21°

Side Length a: 80.0 m

$$\frac{70}{\sin 60^\circ} = \frac{50}{\sin B}$$

$$70 \sin B = 50 \sin 60^\circ$$

$$\sin B = \frac{50 \sin 60^\circ}{70}$$

$$= 0.6186$$

$$\angle B = 38.21^\circ$$

$$\angle A = 180^\circ - 60^\circ - 38.21^\circ = 81.79^\circ$$

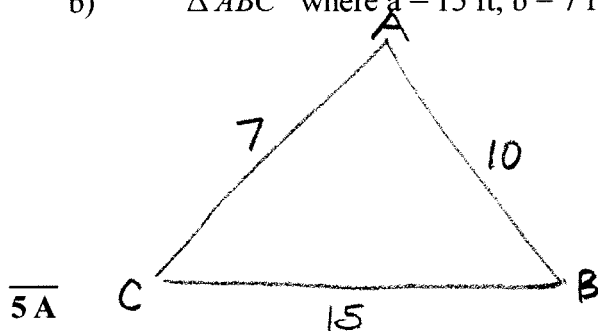
$$\frac{a}{\sin 81.79^\circ} = \frac{70}{\sin 60^\circ}$$

$$a \sin 60^\circ = 70 \sin 81.79^\circ$$

$$a = \frac{70 \sin 81.79^\circ}{\sin 60^\circ}$$

$$= 80.0 \text{ m}$$

b)  $\triangle ABC$  where  $a = 15$  ft,  $b = 7$  ft and  $c = 10$  ft.



Angle A: 122.88°

Angle B: 23.07°

Angle C: 34.05°

$$\cos A = \frac{7^2 + 10^2 - 15^2}{2(7)(10)}$$

$$= -0.5428$$

$$\angle A = 122.88^\circ$$

$$\frac{15}{\sin 122.88^\circ} = \frac{10}{\sin C}$$

$$15 \sin C = 10 \sin 122.88^\circ$$

$$\sin C = \frac{10 \sin 122.88^\circ}{15}$$

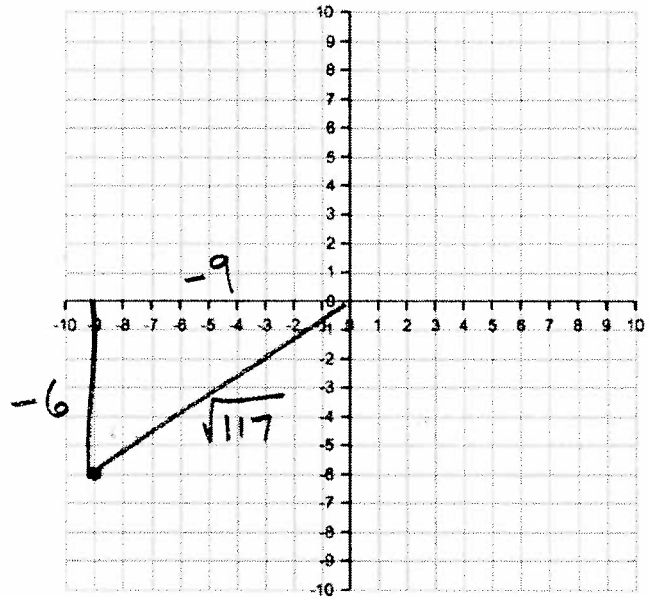
$$\sin C = 0.5599$$

$$\angle C = 34.05^\circ$$

$$\begin{aligned} \angle B &= 180^\circ \\ &- 122.88^\circ \\ &- 34.05^\circ \\ \hline &23.07^\circ \end{aligned}$$

3. What are the exact trigonometric ratios for the angle ( $\theta$ ) represented by a terminal arm that goes through  $(-9, -6)$ ? [Please draw the terminal arm on the grid provided, and label the side lengths. Show any other work in the space below.]

$$\begin{aligned} \text{hyp} &= \sqrt{(-9)^2 + (-6)^2} \\ &= \sqrt{117} \end{aligned}$$



$$\sin \theta = \frac{-6}{\sqrt{117}}$$

$$\cos \theta = \frac{-9}{\sqrt{117}}$$

$$\csc \theta = \frac{\sqrt{117}}{-6}$$

$$\sec \theta = \frac{\sqrt{117}}{-9}$$

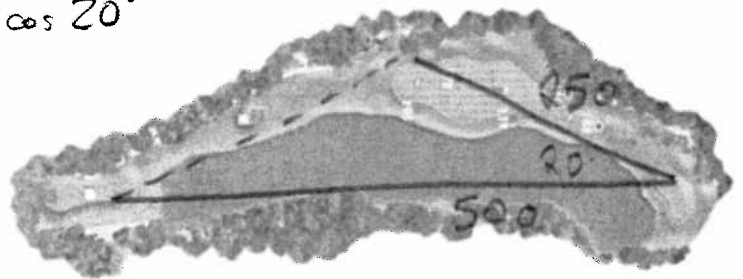
$$\tan \theta = \frac{-6}{-9}$$

$$\cot \theta = \frac{-9}{-6}$$

4. The green on a golf hole lies 500 yards directly **west** of the tee, with a water hazard in between the tee and the green. If a golfer's first shot travels 250 yards from the tee at an angle of  $20^\circ$  [to the north of west] and lands on the fairway, how far will the golfer have to hit the second shot from the fairway in order to land on the green? Include a diagram.

$$\begin{aligned} d^2 &= 500^2 + 250^2 - 2(500)(250)\cos 20^\circ \\ &= 77576.84 \end{aligned}$$

$$d = 278.53 \text{ yards}$$

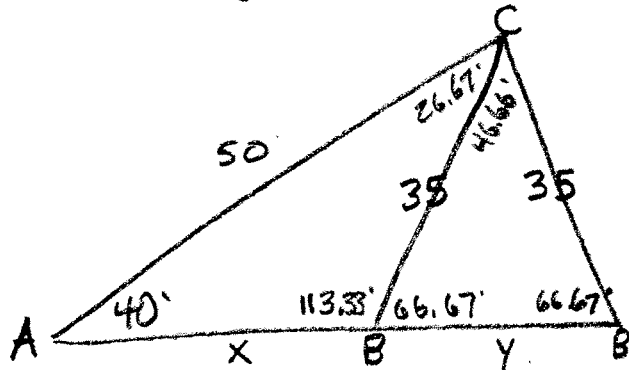


5. Solve **all** triangles that satisfy the given information. Be sure to show **how many** triangles can be formed.

- a)  $a = 35$  m,  $b = 50$  m, and angle  $A = 40^\circ$

$$50 \sin 40^\circ = 32.1$$

$a$  is longer  $\Rightarrow$  2 triangles!



$$\frac{35}{\sin 40^\circ} = \frac{50}{\sin B}$$

$$\sin B = \frac{50 \sin 40^\circ}{35}$$

$$= 0.9183$$

$$\angle B = 66.67^\circ$$

$$180^\circ - 66.67^\circ = 113.33^\circ$$

$$180^\circ - 40^\circ - 113.33^\circ$$

$$= 26.67^\circ$$

$$180^\circ - 66.67^\circ - 66.67^\circ$$

$$= 46.66^\circ$$

$$\frac{X}{\sin 26.67^\circ} = \frac{35}{\sin 40^\circ}$$

$$X = \frac{35 \sin 26.67^\circ}{\sin 40^\circ}$$

$$= 24.44 \text{ m}$$

$$\frac{35}{\sin 66.67^\circ} = \frac{Y}{\sin 46.67^\circ}$$

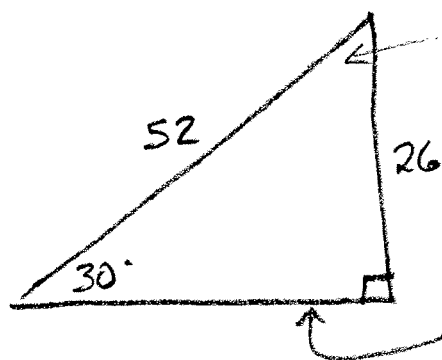
$$Y = \frac{35 \sin 46.67^\circ}{\sin 66.67^\circ}$$

$$= 27.73 \text{ m}$$

9A

- b)  $b = 52$  m,  $a = 26$  m, and angle  $A = 30^\circ$

$$52 \sin 30^\circ = 26 \quad \therefore \text{one triangle}$$



$$180^\circ - 30^\circ - 90^\circ = 60^\circ$$

$$\sqrt{52^2 - 26^2} = 45.03 \text{ m}$$

- c)  $g = 74$  m,  $f = 80$  m, and angle  $G = 70^\circ$

$$80 \sin 70^\circ = 75.2$$

$\therefore$  No triangles

6. Prove each of the following identities.

a)  $\csc x \sec x \cos x \sin x = \sin^2 x + \cos^2 x$

$$LS: \left(\frac{1}{\sin x}\right) \left(\frac{1}{\cos x}\right) \cancel{\cos x} \cancel{\sin x}$$

$$= 1$$

$$\overline{3T} \quad = \sin^2 x + \cos^2 x$$

b)  $\sin^2 x (\csc^2 x - 1) = \cos^2 x$

$$LS: \sin^2 x \left( \frac{1}{\sin^2 x} - 1 \right)$$

$\overline{4T}$

$$= \frac{\sin^2 x}{\sin^2 x} - \sin^2 x$$

$$= 1 - \sin^2 x$$

$$= \cancel{\sin^2 x} + \cos^2 x - \cancel{\sin^2 x}$$

$$= \cos^2 x$$

c)  $\cot x \cos x = \csc x - \sin x$

$$LS: \left(\frac{1}{\tan x}\right) (\cos x)$$

$\overline{4T}$

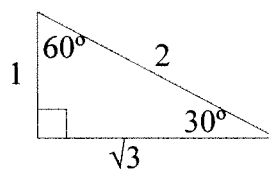
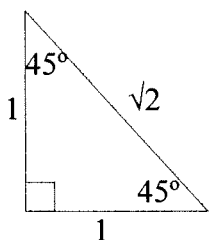
$$= \left(\frac{\cos x}{\sin x}\right) \cos x$$

$$= \frac{\cos^2 x}{\sin x}$$

$$= \frac{1 - \sin^2 x}{\sin x}$$

$$= \frac{1}{\sin x} - \frac{\sin^2 x}{\sin x} = \csc x - \sin x$$

## Reference Material



$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\csc q = \frac{1}{\sin q}$$

$$\sec q = \frac{1}{\cos q}$$

$$\cot q = \frac{1}{\tan q}$$

$$c^2 = a^2 + b^2 - 2ab(\cos C)$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

SOH CAH TOA